

# Nonlinear Spinor Fields in Bianchi type-I spacetime reexamined

Bijan Saha

*Laboratory of Information Technologies*

*Joint Institute for Nuclear Research*

*141980 Dubna, Moscow region, Russia\**

The specific behavior of spinor field in curve space-time with the exception of FRW model almost always gives rise to non-trivial non-diagonal components of the energy-momentum tensor. This non-triviality of non-diagonal components of the energy-momentum tensor imposes some severe restrictions either on the spinor field or on the metric functions. In this paper within the scope of an anisotropic Bianchi type-I Universe we study the role of spinor field in the evolution of the Universe. It is found that there exist two possibilities. In one scenario the initially anisotropic Universe evolves into an isotropic one asymptotically, but in this case the spinor field itself undergoes some severe restrictions. In the second scenario the isotropization takes places almost at the beginning of the process.

PACS numbers: 98.80.Cq

Keywords: Spinor field, dark energy, anisotropic cosmological models, isotropization

---

\* [bijan@jinr.ru](mailto:bijan@jinr.ru); <http://bijansaha.narod.ru>

## I. INTRODUCTION

According to the inflationary scenario, it is believed that a scalar field known as inflaton is responsible for a rapid accelerated expansion of the early Universe [1–3]. For the inflationary mechanism to work there must exist a weakly coupled scalar field which is initially at a false vacuum which leads to the inflation until the right vacuum value is obtained. This inflationary model solves the problem of flatness, isotropy of microwave background radiation and unwanted relics. Contrary to the prediction of the standard cosmological models, recent observations showed an accelerated mode of expansion of the present day Universe [4, 5]. Though the existence of an inflationary scenario is not of much concern, the question of where the scalar field comes from and why it undergoes such a peculiar phase transition from false to right vacuum still remains unanswered. This leads cosmologists to reconsider alternative possibilities.

As one of the way out many specialists considered spinor field as an alternative source. Being related to almost all stable elementary particles such as proton, electron and neutrino, spinor field, especially Dirac spin-1/2 play a principal role at the microlevel. However, in cosmology, the role of spinor field was generally considered to be restricted. Only recently, after some remarkable works by different authors [6–20], showing the important role that spinor fields play on the evolution of the Universe, the situation began to change. This change of attitude is directly related to some fundamental questions of modern cosmology: (i) problem of initial singularity; (ii) problem of isotropization and (iii) late time acceleration of the Universe.

**(i) Problem of initial singularity:** One of the problems of modern cosmology is the presence of initial singularity, which means the finiteness of time. The main purpose of introducing a nonlinear term in the spinor field Lagrangian is to study the possibility of the elimination of initial singularity. In a number of papers [8–12] it was shown that the introduction of spinor field with a suitable nonlinearity into the system indeed gives rise to singularity-free models of the Universe.

**(ii) problem of isotropization:** Although the Universe seems homogenous and isotropic at present, it does not necessarily mean that it is also suitable for a description of the early stages of the development of the Universe and there are no observational data guaranteeing the isotropy in the era prior to the recombination. In fact, there are theoretical arguments that support the existence of an anisotropic phase that approaches an isotropic one [21]. The observations from Cosmic Background Explorer's differential radiometer have detected and measured cosmic microwave background anisotropies in different angular scales. These anisotropies are supposed to hide in their fold the entire history of cosmic evolution dating back to the recombination era and are being considered as indicative of the geometry and the content of the universe. More about cosmic microwave background anisotropy is expected to be uncovered by the investigations of microwave anisotropy probe. There is widespread consensus among the cosmologists that cosmic microwave background anisotropies in small angular scales have the key to the formation of discrete structure. It was found that the introduction of nonlinear spinor field accelerates the isotropization process of the initially anisotropic Universe [10, 11, 13].

**(iii) late time acceleration of the Universe:** Some recent experiments detected an accelerated mode of expansion of the Universe [4, 5]. Detection and further experimental reconfirmation of current cosmic acceleration pose to cosmology a fundamental task of identifying and revealing the cause of such phenomenon. This fact can be reconciled with the theory if one assumes that the Universe is mostly filled with so-called dark energy. This form of matter (energy) is not observable in laboratory and it does not interact with electromagnetic radiation. These facts played decisive role in naming this object. In contrast to dark matter, dark energy is uniformly distributed over the space, does not intertwine under the influence of gravity in all scales and it has a strong negative pressure of the order of energy density. Based on these properties, cosmologists have suggested a number of dark energy models those are able to explain the current accelerated phase of expansion of the Universe. In this connection a series of papers appeared recently in the literature, where a spinor field was considered as an alternative model for dark energy [14–16, 18].

It should be noted that most of the works mentioned above were carried out within the scope of Bianchi type-I cosmological model. Results obtained using a spinor field as a source of Bianchi type-I cosmological field can be summed up as follows: A suitable choice of spinor field nonlinearity

- (i) *accelerates the isotropization process* [10, 11, 13];

- (ii) *gives rise to a singularity-free Universe* [10–13];
- (iii) *generates late time acceleration* [14–16, 18, 19].

Given the role that spinor field can play in the evolution of the Universe, question that naturally pops up is, if the spinor field can redraw the picture of evolution caused by perfect fluid and dark energy, is it possible to simulate perfect fluid and dark energy by means of a spinor field? Affirmative answer to this question was given in the a number of papers [22–26]. In those papers spinor description of matter such as perfect fluid and dark energy was given and the evolution of the Universe given by different Bianchi models was thoroughly studied. In almost all the papers the spinor field was considered to be time-dependent functions and its energy-momentum tensor was given by the diagonal elements only. Some latest study shows that due to the specific connection with gravitational field the energy-momentum tensor of the spinor field possesses non-trivial non-diagonal components as well. In this paper we study the role of non-diagonal components of the energy-momentum tensor of the spinor field in the evolution of the Universe. To our knowledge such study was never done previously. In section II we give the spinor field Lagrangian in details. In section III the system of Einstein-Dirac equations is solved for BI metric without engaging the non-diagonal components of energy-momentum tensor as it was done in previous works of many authors. In section IV we analyze the role of non-diagonal components of energy-momentum tensor on the evolution of the Universe.

## II. SPINOR FIELD LAGRANGIAN

For a spinor field  $\psi$ , the symmetry between  $\psi$  and  $\bar{\psi}$  appears to demand that one should choose the symmetrized Lagrangian [27]. Keeping this in mind we choose the spinor field Lagrangian as [10]:

$$L = \frac{i}{2} \left[ \bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - m_{\text{sp}} \bar{\psi} \psi - F, \quad (2.1)$$

where the nonlinear term  $F$  describes the self-interaction of a spinor field and can be presented as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field. Since  $\psi$  and  $\psi^*$  (complex conjugate of  $\psi$ ) have four component each, one can construct  $4 \times 4 = 16$  independent bilinear combinations. They are

$$S = \bar{\psi} \psi \quad (\text{scalar}), \quad (2.2a)$$

$$P = i \bar{\psi} \gamma^5 \psi \quad (\text{pseudoscalar}), \quad (2.2b)$$

$$v^\mu = (\bar{\psi} \gamma^\mu \psi) \quad (\text{vector}), \quad (2.2c)$$

$$A^\mu = (\bar{\psi} \gamma^5 \gamma^\mu \psi) \quad (\text{pseudovector}), \quad (2.2d)$$

$$T^{\mu\nu} = (\bar{\psi} \sigma^{\mu\nu} \psi) \quad (\text{antisymmetric tensor}), \quad (2.2e)$$

where  $\sigma^{\mu\nu} = (i/2)[\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu]$ . Invariants, corresponding to the bilinear forms, are

$$I = S^2, \quad (2.3a)$$

$$J = P^2, \quad (2.3b)$$

$$I_v = v_\mu v^\mu = (\bar{\psi} \gamma^\mu \psi) g_{\mu\nu} (\bar{\psi} \gamma^\nu \psi), \quad (2.3c)$$

$$I_A = A_\mu A^\mu = (\bar{\psi} \gamma^5 \gamma^\mu \psi) g_{\mu\nu} (\bar{\psi} \gamma^5 \gamma^\nu \psi), \quad (2.3d)$$

$$I_T = T_{\mu\nu} T^{\mu\nu} = (\bar{\psi} \sigma^{\mu\nu} \psi) g_{\mu\alpha} g_{\nu\beta} (\bar{\psi} \sigma^{\alpha\beta} \psi). \quad (2.3e)$$

According to the Fierz identity, among the five invariants only  $I$  and  $J$  are independent as all others can be expressed by them:  $I_v = -I_A = I + J$  and  $I_T = I - J$ . Therefore, we choose the nonlinear term  $F$  to be the function of  $I$  and  $J$  only, i.e.,  $F = F(I, J)$ , thus claiming that it

describes the nonlinearity in its most general form. Indeed, without losing generality we can choose  $F = F(K)$ , with  $K = \{I, J, I+J, I-J\}$ . Here  $\nabla_\mu$  is the covariant derivative of spinor field:

$$\nabla_\mu \psi = \frac{\partial \psi}{\partial x^\mu} - \Gamma_\mu \psi, \quad \nabla_\mu \bar{\psi} = \frac{\partial \bar{\psi}}{\partial x^\mu} + \bar{\psi} \Gamma_\mu, \quad (2.4)$$

with  $\Gamma_\mu$  being the spinor affine connection. In (2.1)  $\gamma$ 's are the Dirac matrices in curve space-time and obey the following algebra

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad (2.5)$$

and are connected with the flat space-time Dirac matrices  $\tilde{\gamma}$  in the following way

$$g_{\mu\nu}(x) = e_\mu^a(x) e_\nu^b(x) \eta_{ab}, \quad \gamma_\mu(x) = e_\mu^a(x) \tilde{\gamma}_a, \quad (2.6)$$

where  $\eta_{ab} = \text{diag}(1, -1, -1, -1)$  and  $e_\mu^a$  is a set of tetrad 4-vectors. The spinor affine connection matrices  $\Gamma_\mu(x)$  are uniquely determined up to an additive multiple of the unit matrix by the equation

$$\nabla_\mu \gamma_\nu = \frac{\partial \gamma_\nu}{\partial x^\mu} - \Gamma_{\nu\mu}^\rho \gamma_\rho - \Gamma_\mu \gamma_\nu + \gamma_\nu \Gamma_\mu = 0, \quad (2.7)$$

with the solution

$$\Gamma_\mu = \frac{1}{4} \tilde{\gamma}_a \gamma^\nu \partial_\mu e_\nu^{(a)} - \frac{1}{4} \gamma_\rho \gamma^\nu \Gamma_{\mu\nu}^\rho, \quad (2.8)$$

Varying (2.1) with respect to  $\bar{\psi}(\psi)$  one finds the spinor field equations:

$$\iota \gamma^\mu \nabla_\mu \psi - m_{\text{sp}} \psi - 2F_K (SK_I + \iota PK_J \gamma^5) \psi = 0, \quad (2.9a)$$

$$\iota \nabla_\mu \bar{\psi} \gamma^\mu + m_{\text{sp}} \bar{\psi} + 2F_K \bar{\psi} (SK_I + \iota PK_J \gamma^5) = 0. \quad (2.9b)$$

Here we denote  $F_K = dF/dK$ ,  $K_I = dK/dI$  and  $K_J = dK/dJ$ .

We consider the case when the spinor field depends on  $t$  only. In this case for the components of energy-momentum tensor we find

$$T_0^0 = m_{\text{sp}} S + F(K), \quad (2.10a)$$

$$T_1^1 = T_2^2 = T_3^3 = F(K) - 2KF_K. \quad (2.10b)$$

The energy-momentum tensor of the spinor field is given by

$$T_\mu^\rho = \frac{i}{4} g^{\rho\nu} \left( \bar{\psi} \gamma_\mu \nabla_\nu \psi + \bar{\psi} \gamma_\nu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma_\nu \psi - \nabla_\nu \bar{\psi} \gamma_\mu \psi \right) - \delta_\mu^\rho L_{\text{sp}} \quad (2.11)$$

where  $L_{\text{sp}}$  in view of (2.9) can be rewritten as

$$\begin{aligned} L_{\text{sp}} &= \frac{i}{2} [\bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi] - m_{\text{sp}} \bar{\psi} \psi - F(K) \\ &= \frac{i}{2} \bar{\psi} [\gamma^\mu \nabla_\mu \psi - m_{\text{sp}} \psi] - \frac{i}{2} [\nabla_\mu \bar{\psi} \gamma^\mu + m_{\text{sp}} \bar{\psi}] \psi - F(K), \\ &= 2F_K (IK_I + JK_J) - F = 2KF_K - F(K). \end{aligned} \quad (2.12)$$

Let us now recall that in the unified nonlinear spinor theory of Heisenberg, the massive term remains absent, and according to Heisenberg, the particle mass should be obtained as a result of quantization of spinor prematter [28, 29]. In the nonlinear generalization of classical field equations, the massive term does not possess the significance that it possesses in the linear one, as it by no means defines total energy (or mass) of the nonlinear field system. Moreover, it was established that only a massless spinor field with the Lagrangian (2.1) describes perfect fluid from

phantom to ekpyrotic matter [22–26]. Thus without losing the generality we can consider the massless spinor field putting  $m = 0$ .

Inserting (2.10a) and (2.10b) into the barotropic equation of state

$$p = W\varepsilon, \quad (2.13)$$

where  $W$  is a constant, one finds

$$2KF_K = (1 + W)F(K), \quad (2.14)$$

with the solution

$$F(K) = \lambda K^{(1+W)/2}, \quad \lambda = \text{const.} \quad (2.15)$$

Depending on the value of  $W$  (2.13) describes perfect fluid from phantom to ekpyrotic matter, namely

$$W = 0, \quad (\text{dust}), \quad (2.16a)$$

$$W = 1/3, \quad (\text{radiation}), \quad (2.16b)$$

$$W \in (1/3, 1), \quad (\text{hard Universe}), \quad (2.16c)$$

$$W = 1, \quad (\text{stiff matter}), \quad (2.16d)$$

$$W \in (-1/3, -1), \quad (\text{quintessence}), \quad (2.16e)$$

$$W = -1, \quad (\text{cosmological constant}), \quad (2.16f)$$

$$W < -1, \quad (\text{phantom matter}), \quad (2.16g)$$

$$W > 1, \quad (\text{ekpyrotic matter}). \quad (2.16h)$$

In account of it the spinor field Lagrangian now reads

$$L_{\text{sp}} = \frac{i}{2} \left[ \bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - \lambda K^{(1+W)/2}. \quad (2.17)$$

Thus a massless spinor field with the Lagrangian (2.17) describes perfect fluid from phantom to ekpyrotic matter. Here the constant of integration  $\lambda$  can be viewed as constant of self-coupling. A detailed analysis of this study was given in [22–25].

A Chaplygin gas is usually described by a equation of state

$$p = -A/\varepsilon^\gamma. \quad (2.18)$$

Then in case of a massless spinor field for  $F$  one finds

$$\frac{F^\gamma dF}{F^{1+\gamma} - A} = \frac{1}{2} \frac{dK}{K}, \quad (2.19)$$

with the solution [23–25]

$$F = (A + \lambda K^{(1+\gamma)/2})^{1/(1+\gamma)}. \quad (2.20)$$

The Spinor field Lagrangian in this case takes the form

$$L_{\text{sp}} = \frac{i}{2} \left[ \bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - (A + \lambda K^{(1+\gamma)/2})^{1/(1+\gamma)}. \quad (2.21)$$

Finally, it should be noted that a quintessence with a modified equation of state

$$p = W(\varepsilon - \varepsilon_{\text{cr}}), \quad W \in (-1, 0), \quad (2.22)$$

where  $\epsilon_{\text{cr}}$  some critical energy density, the spinor field nonlinearity takes the form

$$F = \lambda S^{1+W} + \frac{W}{1+W} \epsilon_{\text{cr}}. \quad (2.23)$$

The spinor field Lagrangian in this case reads

$$L_{\text{sp}} = \frac{i}{2} \left[ \bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - \lambda K^{(1+W)/2} - \frac{W}{1+W} \epsilon_{\text{cr}}. \quad (2.24)$$

Setting  $\epsilon_{\text{cr}} = 0$  one gets (2.17). The purpose of introducing the modified EoS was to avoid the problem of eternal acceleration.

A detailed study of nonlinear spinor field was carried out in [10, 11, 13]. In what follows, exploiting the equation of states we find the concrete form of  $F$  which describes various types of perfect fluid and dark energy.

### III. BIANCHI TYPE-I ANISOTROPIC COSMOLOGICAL MODEL

Let us study the evolution of the Universe filled with spinor field. In doing so we consider the case when the gravitational field is given by an anisotropic Bianchi type-I cosmological model.

Bianchi type-I (BI) model is the simplest anisotropic cosmological model and gives an excellent scope to take into account the initial anisotropy of the Universe. Given the importance of BI model to study the effects of initial anisotropy in the evolution of the Universe, we study this models in details.

We consider the BI metric in the form

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2, \quad (3.1)$$

with  $a$ ,  $b$  and  $c$  being the functions of time only.

For further purpose we define the volume scale  $V$  of the BI metric as

$$V = abc. \quad (3.2)$$

The system of Einstein equations in this case reads

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = \kappa T_1^1, \quad (3.3a)$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} = \kappa T_2^2, \quad (3.3b)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} = \kappa T_3^3, \quad (3.3c)$$

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} = \kappa T_0^0. \quad (3.3d)$$

Here  $T_\mu^\nu$  is the energy momentum tensor of the spinor field.

Solving the Einstein equation on account of the fact that  $T_1^1 = T_2^2 = T_3^3$  for the metric functions one finds [10]

$$a_i = D_i V^{1/3} \exp\left(X_i \int \frac{dt}{V}\right), \quad \prod_{i=1}^3 D_i = 1, \quad \sum_{i=1}^3 X_i = 0, \quad (3.4)$$

with  $D_i$  and  $X_i$  being the integration constants. Thus we see that the metric functions can be expressed in terms of  $V$ .

Summation of (3.3a), (3.3b), (3.3c) and 3 times (3.3d) leads to the equation for  $V$  [10]

$$\ddot{V} = \frac{3\kappa}{2}(T_0^0 + T_1^1)V. \quad (3.5)$$

As we have already found, the components of energy momentum tensor are the function of  $K$ . If  $K$  is a function of  $V$ , then the Eq. (3.5) possesses exact solution. In order to show that  $K$  is a function of  $V$  we go back to the spinor field equations. From (2.9) one dully finds

$$\dot{S}_0 - 4F_K P K_J A_0 = 0, \quad (3.6a)$$

$$\dot{P}_0 + 4F_K S K_I A_0 = 0, \quad (3.6b)$$

$$\dot{A}_0 - 4F_K S K_I P_0 + 4F_K P K_J S_0 = 0, \quad (3.6c)$$

where  $S_0 = SV$ ,  $P_0 = PV$ ,  $A_0 = AV$  with  $A = \bar{\psi}\gamma^0\gamma^5\psi$ . Summation of (3.6a), (3.6b) and (3.6c) leads to

$$S^2 + P^2 + A^2 = C_1/V^2, \quad C_1 = \text{const.} \quad (3.7)$$

On the other hand from (3.6a) and (3.6b) one finds

$$K_I S_0 \dot{S}_0 + K_J P_0 \dot{P}_0 = 0. \quad (3.8)$$

In case of  $K = I$ , i.e.,  $K_I = 1$  and  $K_J = 0$  from (3.8) one finds

$$K = I = S^2 = C_I/V^2, \quad C_I = \text{const.} \quad (3.9)$$

For  $K = J$ , i.e.,  $K_I = 0$  and  $K_J = 1$  from (3.8) one finds

$$K = J = P^2 = C_J/V^2, \quad C_J = \text{const.} \quad (3.10)$$

If  $K = I + J$ , i.e.,  $K_I = 1$  and  $K_J = 1$  from (3.8) one finds

$$K = I + J = S^2 + P^2 = C_{I+J}/V^2, \quad C_{I+J} = \text{const.} \quad (3.11)$$

and finally, for  $K = I - J$ , i.e.,  $K_I = 1$  and  $K_J = -1$  from (3.8) one finds

$$K = I - J = S^2 - P^2 = C_{I-J}/V^2, \quad C_{I-J} = \text{const.} \quad (3.12)$$

Thus we see that for the BI spacetime given by (3.1) one finds

$$K = \frac{V_0^2}{V^2}, \quad V_0 = \text{const.} \quad (3.13)$$

In case of (2.17) we have

$$T_0^0 = \varepsilon = \lambda K^{(1+W)/2}, \quad (3.14a)$$

$$T_1^1 = -p = -W\varepsilon = -W\lambda K^{(1+W)/2}. \quad (3.14b)$$

Eq. (3.5) then takes the form

$$\ddot{V} = \frac{3\kappa}{2}\lambda V_0^{1+W}(1-W)V^{-W}, \quad (3.15)$$

with the solution in quadrature

$$\int \frac{dV}{\sqrt{3\kappa\lambda V_0^{1+W}V^{1-W} + C_1}} = t + t_0. \quad (3.16)$$

Here  $C_1$  and  $t_0$  are the integration constants.

Let us consider the case when the spinor field is given by the Lagrangian (2.21). In this case we have

$$T_0^0 = \varepsilon = (A + \lambda K^{(1+\gamma)/2})^{1/(1+\gamma)}, \quad (3.17a)$$

$$T_1^1 = -p = A/\varepsilon^\gamma = A/(A + \lambda K^{(1+\gamma)/2})^{\gamma/(1+\gamma)}. \quad (3.17b)$$

The equation for  $V$  now reads

$$\ddot{V} = \frac{3\kappa}{2} \left[ (AV^{1+\gamma} + \lambda V_0^{1+\gamma})^{1/(1+\gamma)} + AV^{1+\gamma}/(AV^{1+\gamma} + \lambda V_0^{1+\gamma})^{\gamma/(1+\gamma)} \right], \quad (3.18)$$

with the solution

$$\int \frac{dV}{\sqrt{C_1 + 3\kappa V (AV^{1+\gamma} + \lambda V_0^{1+\gamma})^{1/(1+\gamma)}}} = t + t_0, \quad C_1 = \text{const.} \quad t_0 = \text{const.} \quad (3.19)$$

Inserting  $\gamma = 1$  we come to the result obtained in [30].

Finally we consider the case with modified quintessence. Taking into account that

$$T_0^0 = \lambda K^{(1+W)/2} + \frac{W}{1+W} \varepsilon_{\text{cr}}, \quad (3.20a)$$

$$T_1^1 = T_2^2 = T_3^3 = -\lambda W K^{(1+W)/2} + \frac{W}{1+W} \varepsilon_{\text{cr}}, \quad (3.20b)$$

for  $V$  in this case we find

$$\ddot{V} = \frac{3\kappa}{2} \left[ \lambda V_0^{1+W} (1-W) V^{-W} + 2W \varepsilon_{\text{cr}} V / (1+W) \right], \quad (3.21)$$

with the solution in quadrature

$$\int \frac{dV}{\sqrt{3\kappa [\lambda V_0^{1-W} V^{1-W} + W \varepsilon_{\text{cr}} V^2 / (1+W)] + C_1}} = t + t_0. \quad (3.22)$$

Here  $C_1$  and  $t_0$  are the integration constants. Comparing (3.22) with those with a negative  $\Lambda$ -term we see that  $\varepsilon_{\text{cr}}$  plays the role of a negative cosmological constant.

Let us also write the components of the spinor field explicitly. For simplicity, we consider the case when  $K = I$ . Defining  $\mathcal{D} = 2SF_K K_I = 2SF_K$  in this case we find

$$\psi_1(t) = (C_1/\sqrt{V}) \exp \left( -i \int \mathcal{D} dt \right), \quad (3.23a)$$

$$\psi_2(t) = (C_2/\sqrt{V}) \exp \left( -i \int \mathcal{D} dt \right), \quad (3.23b)$$

$$\psi_3(t) = (C_3/\sqrt{V}) \exp \left( i \int \mathcal{D} dt \right), \quad (3.23c)$$

$$\psi_4(t) = (C_4/\sqrt{V}) \exp \left( i \int \mathcal{D} dt \right), \quad (3.23d)$$



with  $C_1, C_2, C_3, C_4$  being the integration constants and related to  $V_0$  as

$$C_1^* C_1 + C_2^* C_2 - C_3^* C_3 - C_4^* C_4 = V_0.$$

In what follows we will study the obtained results within the scope of some recent findings, namely the fact that the spinor field possesses non-trivial non-diagonal components of the energy-momentum tensor.

#### IV. WHAT'S WRONG?

In first view everything looks good and the papers written till the date on this subject seems correct. But there is still something to be worried about. In what follows, we speak about the new findings on this field.

It should be remembered that the spinor field is more sensitive to the gravitational one. It is due to specific spinor connection in curve space-time. So, let us first write the spin affine connection explicitly. For BI metric it looks:

$$\Gamma_0 = 0, \quad \Gamma_i = \frac{\dot{a}_i}{a} \tilde{\gamma}^i \tilde{\gamma}^0, \quad (4.1)$$

Taking it into account from (2.11) it can be easily verified that the energy-momentum tensor of the spinor field possesses non-trivial non-diagonal components as well.

$$T_0^0 = m_{\text{sp}} S + F(K) \equiv F(K), \quad (4.2a)$$

$$T_1^1 = T_2^2 = T_3^3 = 2KF_K - F(K), \quad (4.2b)$$

$$T_2^1 = \frac{\iota}{4} \frac{a_2}{a_1} \left( \frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right) \bar{\psi} \tilde{\gamma}^1 \tilde{\gamma}^2 \tilde{\gamma}^0 \psi, \quad (4.2c)$$

$$T_3^1 = \frac{\iota}{4} \frac{a_3}{a_1} \left( \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right) \bar{\psi} \tilde{\gamma}^3 \tilde{\gamma}^1 \tilde{\gamma}^0 \psi, \quad (4.2d)$$

$$T_3^2 = \frac{\iota}{4} \frac{a_3}{a_2} \left( \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} \right) \bar{\psi} \tilde{\gamma}^2 \tilde{\gamma}^3 \tilde{\gamma}^0 \psi. \quad (4.2e)$$

So the complete set of Einstein equation for BI metric should be

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = \kappa(2KF_K - F(K)), \quad (4.3a)$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} = \kappa(2KF_K - F(K)), \quad (4.3b)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} = \kappa(2KF_K - F(K)), \quad (4.3c)$$

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} = \kappa(m_{\text{sp}} S + F(K)) \equiv \kappa F(K), \quad (4.3d)$$

$$0 = \frac{\iota}{4} \frac{a_2}{a_1} \left( \frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right) \bar{\psi} \tilde{\gamma}^1 \tilde{\gamma}^2 \tilde{\gamma}^0 \psi, \quad (4.3e)$$

$$0 = \frac{\iota}{4} \frac{a_3}{a_1} \left( \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right) \bar{\psi} \tilde{\gamma}^3 \tilde{\gamma}^1 \tilde{\gamma}^0 \psi, \quad (4.3f)$$

$$0 = \frac{\iota}{4} \frac{a_3}{a_2} \left( \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} \right) \bar{\psi} \tilde{\gamma}^2 \tilde{\gamma}^3 \tilde{\gamma}^0 \psi. \quad (4.3g)$$

The equations (4.3e), (4.3f) and (4.3g) impose some severe restrictions either on the spinor field, or on the metric functions, or on both of them.

If the restrictions are imposed on the spinor field, we obtain

$$\bar{\psi}\bar{\gamma}^1\bar{\gamma}^2\bar{\gamma}^0\psi = \bar{\psi}\bar{\gamma}^3\bar{\gamma}^1\bar{\gamma}^0\psi = \bar{\psi}\bar{\gamma}^2\bar{\gamma}^3\bar{\gamma}^0\psi = 0. \quad (4.4)$$

In this case the expressions obtained for the metric functions in the earlier papers remain unaffected. But the components of the spinor field will undergo some changes. It should be verified that in this case the integration constants in (3.23) should obey

$$C_1^*C_1 - C_2^*C_2 - C_3^*C_3 + C_4^*C_4 = 0, \quad (4.5a)$$

$$C_1^*C_2 - C_2^*C_1 + C_3^*C_4 - C_4^*C_3 = 0, \quad (4.5b)$$

$$C_1^*C_2 + C_2^*C_1 + C_3^*C_4 + C_4^*C_3 = 0, \quad (4.5c)$$

$$C_1^*C_1 + C_2^*C_2 - C_3^*C_3 - C_4^*C_4 = V_0, \quad (4.5d)$$

which gives

$$C_1^*C_2 + C_3^*C_4 = C_2^*C_1 + C_4^*C_3 = 0, \quad (4.6a)$$

$$C_1^*C_1 - C_3^*C_3 = C_2^*C_2 - C_4^*C_4 = \frac{V_0}{2}. \quad (4.6b)$$

In the cases considered above for volume scale we obtained the expressions given by (3.16), (3.19) and (3.22), for the Universe filled with quintessence, Chaplygin gas and quintessence with modified equation of state, respectively. The Universe in these cases is initially anisotropic which evolves into an isotropic one asymptotically [25, 26].

The other possibility is to keep the components of the spinor field unaltered. In this case from (4.3e), (4.3f) and (4.3g) for the metric functions one immediately finds:

$$\frac{\dot{a}_1}{a_1} = \frac{\dot{a}_2}{a_2} = \frac{\dot{a}_3}{a_3} \equiv \frac{\dot{a}}{a}. \quad (4.7)$$

Taking into account that

$$\frac{\ddot{a}_i}{a_i} = \frac{d}{dt} \left( \frac{\dot{a}_i}{a_i} \right) + \left( \frac{\dot{a}_i}{a_i} \right)^2 = \frac{d}{dt} \left( \frac{\dot{a}}{a} \right) + \left( \frac{\dot{a}}{a} \right)^2 = \frac{\ddot{a}}{a},$$

the system (4.3) can be written as a system of two equations:

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \kappa T_1^1, \quad (4.8a)$$

$$3\frac{\dot{a}^2}{a^2} = \kappa T_0^0. \quad (4.8b)$$

In order to find the solution that satisfies both (4.8a) and (4.8b) we rewrite (4.8a) in view of (4.8b) in the following form:

$$\ddot{a} = \frac{\kappa}{6} (3T_1^1 - T_0^0) a. \quad (4.9)$$

Thus in account of non-diagonal components of the spinor field, we though begin with Bianchi type-I space time, in reality solving the Einstein field equations for FRW model. Before solving the equation (4.9), let us go back to (3.4). Taking into account that

$$\frac{\dot{a}_i}{a_i} = \frac{\dot{V}}{3V} + \frac{X_i}{V}, \quad (4.10)$$

in view of (4.7) we find that

$$X_1 = X_2 = X_3 = 0. \quad (4.11)$$

The triviality of the integration constant  $X_i$  follows from the fact that  $X_1 + X_2 + X_3 = 0$ . Thus the solution (3.4) should be written as

$$a_i = D_i V^{1/3} = D_i a, \quad \prod_{i=1}^3 D_i = 1, \quad (4.12)$$

which means it represents a tiny sector of the general solutions (3.4) which one obtains for the BI model in case of isotropic distribution of matter with trivial non-diagonal components of energy-momentum tensor, e.g., when the Universe is filled with perfect fluid, dark energy etc.

Let us now define  $a = V^{1/3}$  for different cases. In doing so we recall that  $K$  in this case takes the form

$$K = \frac{a_0^6}{a^6}, \quad a_0 = \text{const.} \quad (4.13)$$

Then the equation for  $a$  in case of the spinor field given by (2.17) takes the form

$$\ddot{a} = -\frac{\kappa\lambda}{6}(1+3W)a_0^{3(1+W)}a^{-(2+3W)}, \quad (4.14)$$

with the solution is quadrature

$$\int \frac{da}{\sqrt{(\kappa\lambda/3)a_0^{3(1+W)}a^{-(1+3W)} + E_1}} = t, \quad (4.15)$$

with  $E_1$  being integration constant.

As far as Chaplygin scenario is concerned in this case we have

$$\ddot{a} = -\frac{\kappa}{6} \frac{2Aa^{3(1+\gamma)} - \lambda a_0^{3(1+\gamma)}}{a^2(Aa^{3(1+\gamma)} - \lambda a_0^{3(1+\gamma)})^{\gamma/(1+\gamma)}}. \quad (4.16)$$

This equation can be solved numerically.

Finally we consider the case with modified quintessence. Inserting (3.20a) and (3.20b) into (4.9) in this case we find

$$\ddot{a} = -\frac{\kappa}{6} \left[ (3W+1)\lambda a_0^{3(1+W)}a^{-(3W+2)} - \frac{2W}{1+W}\epsilon_{\text{cr}}a \right], \quad (4.17)$$

with the solution

$$\int \frac{da}{\sqrt{(\kappa/3) \left[ \lambda a_0^{3(1+W)}a^{-(3W+1)} + [W/(1+W)]\epsilon_{\text{cr}}a^2 + E_2 \right]}} = t, \quad E_2 = \text{const.} \quad (4.18)$$

It can be shown that in case of modified quintessence the pressure is sign alternating. As a result we have a cyclic mode of evolution.

In what follows we illustrate the evolution of the Universe filled with quintessence, Chaplygin gas and quintessence with modified equation of state for two different cases: when the restrictions are imposed on the spinor field and when the metric functions were restricted. In Figures 1, 2 and 3 we illustrated the evolution of the Universe filled with quintessence, Chaplygin gas and quintessence with modified equation of state, respectively. The solid (red) line stands for the volume scale, when the restrictions due to non-zero non-diagonal components of the energy momentum tensor of the spinor field, were imposed on the components of the spinor field. In this case the isotropization takes place asymptotically. The blue line shows the evolution of the Universe when due to the non-zero non-diagonal components of the energy momentum tensor of the spinor field leads to the immediate isotropization of the Universe. Here we plot the volume scale as  $a^3$ , which  $a$  being the average scale factor.

As one sees, in case of early isotropization the Universe grows rapidly.

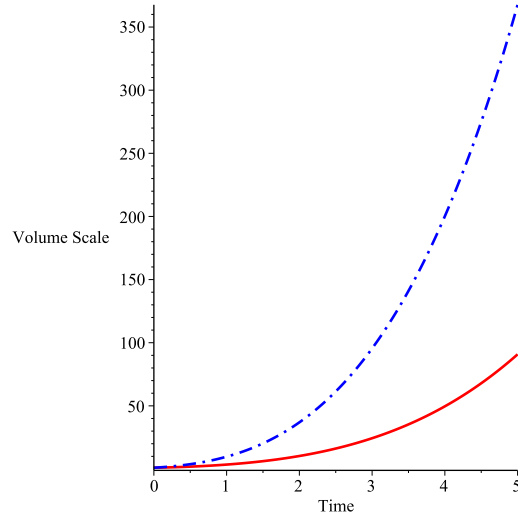


FIG. 1. Evolution of the Universe filled with quintessence. The solid (red) line stands for volume scale  $V$ , while the dash-dot (blue) line stands for  $a^3$ .

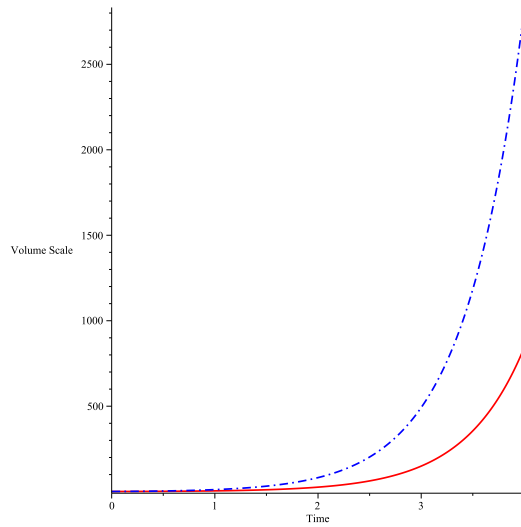


FIG. 2. Evolution of the Universe filled with Chaplygin gas. The solid (red) line stands for volume scale  $V$ , while the dash-dot (blue) line stands for  $a^3$ .

## V. CONCLUSION

Within the scope of Bianchi type-I space time we study the role of spinor field on the evolution of the Universe. It is shown that even in case of space independent of the spinor field it still possesses non-zero non-diagonal components of energy-momentum tensor thanks to its specific relation with gravitational field. This fact plays vital role on the evolution of the Universe. There might be two different scenarios. In one case only the components of the spinor field are affected leaving the space-time initially anisotropic that evolves into an isotropic one asymptotically. According to the second scenario, the space-time becomes isotropic right from the beginning, i.e.,

$$a_1 \sim a_2 \sim a_3, \quad (5.1)$$

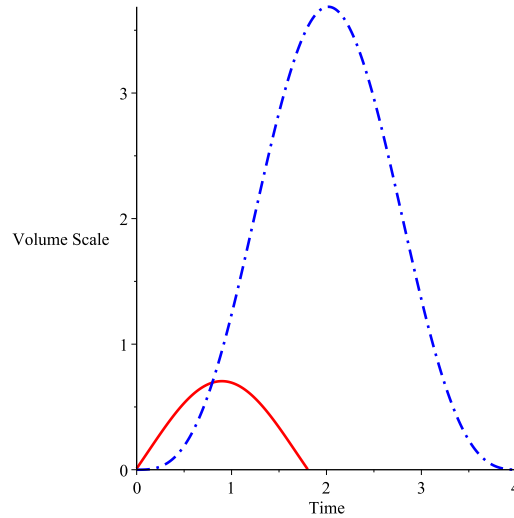


FIG. 3. Evolution of the Universe filled with quintessence with modified equation of state. The solid (red) line stands for volume scale  $V$ , while the dash-dot (blue) line stands for  $a^3$ .

and can be completely described by the Einstein field equations for FRW metric. As numerical analysis shows, in case of early isotropization the Universe expands rather rapidly. There might be another possibility when the non-diagonal components of energy-momentum tensor influence both the spinor field and metric functions simultaneously. We plan to study this last case within the scope of other Bianchi models in our forthcoming papers.

#### Acknowledgments

This work is supported in part by a joint Romanian-LIT, JINR, Dubna Research Project, theme no. 05-6-1060-2005/2013.

- 
- [1] A.H. Guth, *Phys. Rev. D* **23**, 347 (1981)
  - [2] B. Ratra and P.J.E. Peebles, *Phys. Rev. D* **37**, 3406 (1988)
  - [3] K.A. Olive, *Phys. Rep.* **190**, 307 (1990)
  - [4] A.G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998)
  - [5] S Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999)
  - [6] M. Henneaux *Phys. Rev. D* **21**, 857 (1980)
  - [7] U. Ochs and M. Sorg *Int. J. Theor. Phys.* **32**, 1531 (1993)
  - [8] B. Saha and G.N. Shikin *Gen. Relat. Grav.* **29**, 1099 (1997)
  - [9] B. Saha and G.N. Shikin *J Math. Phys.* **38**, 5305 (1997)
  - [10] B. Saha *Phys. Rev. D* **64**, 123501 (2001)
  - [11] B. Saha and T. Boyadjiev *Phys. Rev. D* **69**, 124010 (2004)
  - [12] B. Saha *Phys. Rev. D* **69**, 124006 (2004)
  - [13] B. Saha *Phys. Particle. Nuclei.* **37**, Suppl. 1, S13 (2006)
  - [14] B. Saha *Grav. & Cosmol.* **12**(2-3)(46-47), 215 (2006)
  - [15] B. Saha *Romanian Rep. Phys.* **59**, 649 (2007).
  - [16] B. Saha *Phys. Rev. D* **74**, 124030 (2006)
  - [17] C. Armendáriz-Picón and P.B. Greene *Gen. Relat. Grav.* **35**, 1637 (2003)

- [18] M.O. Ribas, F.P. Devecchi, and G.M. Kremer *Phys. Rev. D* **72**, 123502 (2005)
- [19] R.C de Souza and G.M. Kremer *Class. Quantum Grav.* **25**, 225006 (2008)
- [20] G.M. Kremer and R.C de Souza arXiv:1301.5163v1 [gr-qc]
- [21] C.W. Misner *Astrophys. J.* **151**, 431 (1968)
- [22] V.G.Krechet, M.L. Fel'chenkov, and G.N. Shikin *Grav. & Cosmol.* **14** No 3(55), 292 (2008)
- [23] B. Saha *Cent. Euro. J. Phys.* **8**, 920 (2010a)
- [24] B. Saha *Romanian Rep. Phys.* **62**, 209 (2010b)
- [25] B. Saha *Astrophys. Space Sci.* **331**, 243 (2011)
- [26] B. Saha *Int. J. Theor. Phys.* **51**, 1812 (2012)
- [27] T.W.B. Kibble *J. Math. Phys.* **2**, 212 (1961)
- [28] W. Heisenberg *Phisica* **19**, 897 (1953)
- [29] W. Heisenberg *Rev. Mod. Phys.* **29**, 269 (1957)
- [30] B. Saha *Chinese J. Phys.* **43**(6), 1035 (2005)